ture. Wing leading edges made of two materials, but with small radii, require a solution of the diffusion equation in cylindrical coordinates. The presented η curves may be used for a leading edge problem, but the bondline temperature obtained would be lower by an unknown amount than would actually be experienced with a more nearly exact solution using cylindrical coordinates.

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Fundamental Sloshing Frequency for an Inclined, Fluid-Filled Right Circular Cylinder

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Nomenclature

a_k^m = kth constant in expansion for ψ_m
F = frequency parameter, $\omega^2 R/g$
g = gravitational acceleration
h = depth of fluid in container (Fig. 1)
I_{ij} = $\int_0^{2\pi} (\cos\theta)^{L+i}(\sin\theta)^j d\theta$
L = $m + p - k - q$, where m, p, k, q are summation indices
N = number of terms in expansion for φ
n = outward normal to surface bounding fluid
R = radius of container
r,z = radial and longitudinal coordinates (Fig. 1)
S = undisturbed free surface of fluid
t = time
α_m = mth constant in expansion for φ
θ = angular coordinate (Fig. 1)
ρ = fluid density
τ = region containing fluid
ϕ = angle of inclination (Fig. 1)
φ = velocity potential
ψ_m = mth function in expansion for φ
ω = fundamental sloshing frequency

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STUDIES[1,2] of the natural sloshing frequencies of incompressible fluids have found important applications in the design of space flight vehicles. Previous investigations have been restricted to cases where the container was vertically aligned with the Earth's gravitational field. However, with increasing attention being directed toward the design of shuttle vehicles operating between Earth and orbiting platforms, it becomes necessary to consider the problem of the behavior of fuel in a tank which is inclined to the Earth's gravitational field. The present Note demonstrates how the variational principle first employed by Lawrence, Wang, and Reddy[3] can be used to compute the fundamental frequency of oscillation of a liquid in an arbitrarily inclined container.

The fluid is assumed to be incompressible and inviscid, and the tank is considered to be rigid. The governing equations for small oscillations under these restrictions are as follows:[2] Within the fluid region the velocity potential φ must satisfy the Laplace equation

$$\nabla^2\varphi = 0 \tag{1}$$

while, on the wetted surface,

$$\partial\varphi/\partial n = 0 \tag{2}$$

whereas, on the undisturbed free surface of the liquid,

$$\partial\varphi/\partial n + (1/g)\,\partial^2\varphi/\partial t^2 = 0 \tag{3}$$

It has been shown[3] that the function φ which extremizes the integral

$$J = \frac{1}{2}\rho\int_{t_1}^{t_2}\left[\int_\tau \nabla\varphi\cdot\nabla\varphi d\tau - \left(\frac{1}{g}\right)\int_S\left(\frac{\partial\varphi}{\partial t}\right)^2 dS\right]dt \tag{4}$$

must also satisfy Eqs. (1–3).

Rattayya[4] has employed a series expansion of φ in terms of harmonic polynomial functions to extremize the integral Eq. (4) and thus obtain the natural frequencies of liquid oscillation in a vertical axisymmetric ellipsoidal tank. His approach is modified here to obtain solutions for the case of a skewed cylinder.

Using the coordinate system of Fig. 1, the limits of the

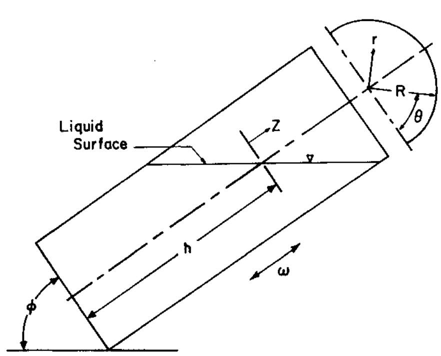

Fig. 1 Schematic of inclined fluid-filled cylinder.

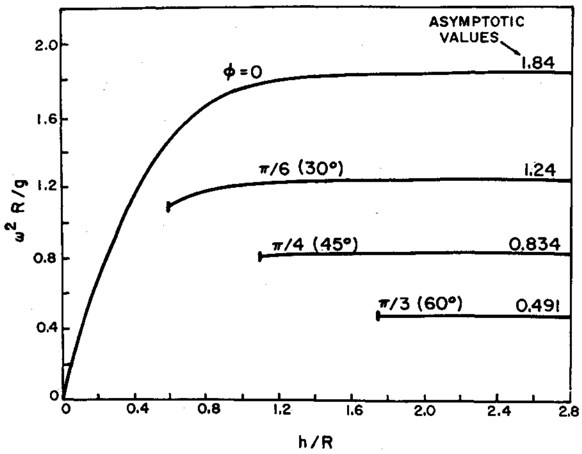

Fig. 2 Predicted variation of sloshing frequency with inclination angle and depth of fluid.

integrals in Eq. (4) are applied to give

$$J = \frac{1}{2}\rho \int_{t_1}^{t_2} \left\{ \int_0^{2\pi}\int_0^R\int_{-h}^{\bar z} \nabla\varphi\cdot\nabla\varphi r dz dr d\theta - (g\cos\phi)^{-1}\int_0^R\int_0^{2\pi} [(\partial\varphi/\partial t)^2]_{z=\bar z}\, r d\theta dr \right\} dt \qquad (5)$$

where $\bar z(r,\theta) = r\tan\phi\cos\theta$ is the distance from the plane $z = 0$ to the undisturbed free surface. It is also assumed, following Rattayya, that φ is expressible in the form

$$\varphi = \sum_{m=1}^{N} \alpha_m R^{-m}\,\psi_m(r,z)\cos\theta\sin\omega t \qquad (6)$$

where α_m are constants, and each of the quantities $\psi_m \cos\theta$ is harmonic. The latter functions may be written as

$$\psi_m(r,z) = \sum_{k=0}^{m} a_k{}^m r^k z^{m-k} \qquad (7)$$

where $a_k{}^m$ are constants determined by substituting Eq. (7) into the Laplace equation $\nabla^2(\psi_m\cos\theta) = 0$.

Substitution of Eqs. (6) and (7) into (5) gives, after integration over 1 cycle in time,

$$J = (\pi\rho R/2\omega)\left[\sum_{m=1}^{N} b_{mm}\alpha_m{}^2 + 2\sum_{m=1}^{N-1}\sum_{p=m+1}^{N} b_{mp}\alpha_m\alpha_p\right] \qquad (8)$$

where

$$b_{mp} = b_{pm} = \sum_{k=1}^{m}\sum_{q=1}^{p} a_k{}^m a_q{}^p \left\{\frac{(\tan\phi)^{L+1}(kqI_{30} + I_{12})}{(L+1)(m+p+1)} - \frac{(-h/R)^{L+1}\pi(1+kq)}{(L+1)(k+q)} + \frac{(m-k)(p-q)}{L-1} \times \left[\frac{(\tan\phi)^{L-1}I_{10}}{m+p+1} - \frac{(-h/R)^{L-1}\pi}{2+k+q}\right] - \frac{F(\tan\phi)^L I_{20}}{(m+p+2)\cos\phi}\right\}$$

The parameters F, L, and I_{ij} are given in the Nomenclature. It is observed that the integral of Eq. (8) will be extremized when $\det[b_{mp}] = 0$.

For a specified number of terms (N) in the expansion for φ and given values of the angle ϕ and the geometric ratio h/R, it is seen that $\det[b_{mp}]$ is a function only of the dimensionless frequency parameter $F = \omega^2 R/g$. The numerical technique employed for determining F was as follows: Beginning with $N = 10$ and $F = 0$ the variation of $\det[b_{mp}]$ was developed until the determinant became zero. The value of N was increased by one and the procedure repeated. This iteration process was continued until successive values of F (for a zero determinant) differed by less than 1%. This degree of accuracy required values of N no larger than 12 for the cases considered. Because of the large order of the determinants involved it was necessary to employ double precision computation on an IBM 360 machine.

Some results of the foregoing computation program are illustrated in Fig. 2. These curves show the variation of $\omega^2 R/g$ with h/R for three nonzero inclination angles in addition to the exact solution[1] for $\phi = 0$ which was also obtained numerically as a check. Solutions were obtained only for those cases where the cylinder bottom was wetted, i.e., $h/R \geq \tan\phi$. Thus only for $\phi = 0$ does the frequency curve extend to the origin. It is also observed that the frequency decreases with increasing ϕ and that asymptotic values are reached in all cases for large h/R.

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Testing of Spacecraft Systems in a Simulated Ionospheric Plasma

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To test high-voltage systems adequately, one must consider the interaction between the charged particles of the ionosphere and the system's high electric fields. Failures and malfunctions in orbit have occurred that have been attributed to an interaction with the plasma.[1] The plasma effects on high-voltage systems include leakage currents, rf attenuation, large sheaths, inducement of outgassing, and possible catastrophic discharging.

Figure 1 shows the variation of charged-particle density vs height in the ionosphere.[2] Note the large day-night variation and the range of densities, 10^4–10^6 cm^{-3}. These charged particle densities produce an ion flux on the order of 10^{11} ions/sec-cm^2 as the result of the relative motion between the ions and spacecraft. Because of the high electron mobility, there is no directed electron flux.

The laboratory simulation is produced by creating a low-level discharge with an electron gun made of a nude ionization gage wired so that an electron beam is accelerated through the grid.[2] This beam produces ionization between two screens in the ion source, which is composed of four parallel screens. An electric field polarizes the plasma so that the ions are accelerated through the other screens into the test volume. The additional screens prevent high-energy-beam electrons from entering the test volume. This ion beam is neutralized by

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